

Exercise 5.2

Question 1.

Classify the following sets into empty set, finite set and infinite set. In case of (non-empty) finite sets, mention the cardinal number.

- (i) {all colours of a rainbow}
- (ii) $\{x \mid x \text{ is a prime number between 7 and 11}\}$
- (iii) {multiples of 5}
- (iv) {all straight lines drawn in a plane}
- (v) $\{x \mid x \text{ is a digit in the numeral 550131527}\}$
- (vi) $\{x \mid x \text{ is a letter in word SUFFICIENT}\}$
- (vii) $\{x \mid x = 4n, n \in I \text{ and } x < 10\}$
- (viii) $\{x \mid x \in N, x \text{ is a prime factor of 180}\}$
- (ix) $\{x : x \text{ is a vowel in the word WHY}\}$
- (x) $\{x : x = 5n, n \in W \text{ and } x < 60\}$

Solution:

- (i) It is a finite set having 7 elements.
- (ii) It is an empty set.
- (iii) It is an infinite set having unlimited elements
- (iv) It is an infinite set having unlimited number of elements.
- (v) It is a finite set having 6 elements i.e., 0, 1, 2, 3, 5, 7.
- (vi) It is a finite set having 8 elements i.e., S, U, F, I, C, E, N, T.
- (vii) It is an infinite set having the set of integers
i.e., unlimited number of elements.
- (viii) It is a finite set having 3 elements.
- (ix) $\{x : x \text{ is a vowel in the word WHY}\}$
It is an empty set as there is no vowel in the word why
- (x) $\{x : x = 5n, n \in W \text{ and } x < 60\}$
 $= \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55\}$
It is finite set and it has 12 elements.

Question 2.

Which of the following describe the same sets:

- (i) {vowels of English alphabet} and {e, a, u, i, o}
- (ii) {a, b, d} and {d, a, b, b}
- (iii) {letters of PUPPET} and {E, T, P, U}
- (iv) {1, 2, 3} and {2, 3, 4}

(v) $\{1, 2, 3, 4, 5\}$ and $\{x \mid x \in \mathbb{N}, x \leq 5\}$

Solution:

- (i) the given sets are the same sets.
- (ii) These are the same sets.
- (iii) The given sets are the same sets.
- (iv) These are not the same sets.
- (v) The given sets are the same sets.

Question 3.

Find pairs/groups of equal sets from the following sets:

$A = \{0, 1, 2, 3\}$

$B = \{x : x^2 < 10, x \in \mathbb{W}\}$

$C = \{\text{letters of word FOLLOW}\}$

$D = \{\text{days of a week}\}$

$E = \{x \mid x \in \mathbb{W}, x < 4\}$

$F = \{\text{letters of word FLOW}\}$

$G = \{\text{Monday, Tuesday, } \dots, \text{ Sunday}\}$

$H = \{\text{letters of word WOLF}\}$

Solution:

$A = B = E$ because if we write B and E in tabular form, we get the same elements.

$C = F = H$ because the elements in a set can be rearranged as each set can be written as $\{F, O, L, W\}$ form.

$D = G$ because if we write D in tabular we get the same elements.

Question 4.

Find pairs/groups of equivalent sets from the following sets.

$A = \{\text{colours of a rainbow}\}$

$B = \{\text{letters of word GOOD}\}$

$C = \{x : x \text{ is a digit in the numeral } 371011489\}$

$D = \{\text{letters of word TOM}\}$

$E = \{x : x \in \mathbb{I}, x^2 < 10\}$

$F = \{\text{months of a year}\}$

$G = \{\text{days of a week}\}$

$H = \{x \mid x = 3n, n \in \mathbb{W} \text{ and } n < 12\}$

$I = \{\text{all even numbers between 1 and 53}\}$

$J = \{\text{all letters of English alphabets}\}$

Solution:

A, C, E and G are equivalent sets as these
all have same number of elements i.e, 7 elements.

$B \Leftrightarrow D$ as $n(B) = 3 = n(D)$

$F \Leftrightarrow H$ as $n(F) = 12 = n(H)$

$I \Leftrightarrow J$ as $n(I) = 26 = n(J)$

Question 5.

In the following, find whether $A \subset B$ or $B \subset A$ or none of these:

(i) $A = \{1, 2, 3\}$, $B = \{2, 3, 3, 3, 1, 3\}$

(ii) $A = \{2, 4, 6, \dots\}$, $B = \{\text{all natural numbers}\}$

(iii) $A = \{x \mid x \in I, x^2 < 20\}$, $B = \{0, 1, 2, 3, 4\}$

(iv) $A = \{\text{letters of KING}\}$,

$B = \{\text{letters of QUEEN}\}$

Solution:

In the following, find whether

$A \subset B$ or $B \subset A$ or none of these

(i) $A = \{1, 2, 3\}$, $B = \{2, 3, 3, 3, 1, 3\}$

$= \{2, 3, 1\}$

$A \subset B$ and $B \subset A$; i.e., $A = B$

(ii) $A = \{2, 4, 6, \dots\}$, $B = \{\text{all natural numbers}\}$

$= \{1, 2, 3, 4, 5, 6, 7, \dots\}$

$A \subset B$ but $B \not\subset A$

(iii) $A = \{x \mid x \in I, x^2 < 20\}$, $B = \{0, 1, 2, 3, 4\}$

$= \{0, 1, 4, 9, 16\}$

$= \{(0)^2, (1)^2, (2)^2, (3)^2, (4)^2\}$

$B = \{0, 1, 2, 3, 4\}$

$B \subset A$ but $A \not\subset B$

(iv) $A = \{\text{letters of KING}\} = \{K, I, N, G\}$

$B = \{\text{letters of QUEEN}\} = \{Q, U, E, N\}$

Here, $A \not\subset B$ and $B \not\subset A$

Neither $A \subset B$ nor $B \subset A$

Question 6.

State whether each of the following statement is true or false for the sets A and B where

$A = \{\text{letters of CLOUD}\}$ and $B = \{\text{letters of KOLKATA}\}$

(i) $A \subset B$

(ii) $B \subset A$

(iii) $A \leftrightarrow B$

Solution:

$A = \{\text{letters of CLOUD}\} = \{C, L, O, U, D\}$

$B = \{\text{Letters of KOLKATA}\} = \{K, O, L, A, T\}$

(i) $A \subset B$: It is false because some elements of A are not the elements of B.

(ii) $B \subset A$: It is false because some elements of B is not a member of A.

(iii) $A \leftrightarrow B$. It is true as $n(A) = 5 = n(B)$.

Question 7.

Write all the subsets of the following sets:

(i) Φ

(ii) $\{3, 5\}$

(iii) $\{2, 4, 6\}$

Solution:

(i) Subset of Φ is Φ

(ii) Empty set is a subset of every set so,
the subsets are $\Phi, \{3\}, \{5\}, \{3, 5\}$.

(iii) Empty set is a subset of every set.

So the subsets are $\Phi, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{4, 6\}, \{2, 6\}, \{2, 4, 6\}$

Question 8.

If $A = \{x : x = 2n, n < 5\}$, then find A when

(i) $\xi = N$

(ii) $\xi = W$

(iii) $\xi = I$

Solution:

(i) Natural numbers less than 5 are, 1, 2, 3, 4.

Given $x = 2n$, putting $n = 1, 2, 3, 4$, we get,

$$x = 2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4$$

$$= 2, 4, 6, 8.$$

The given set can be written as $\{2, 4, 6, 8\}$

(Every set is a subset of universal set i.e., $A \subset \xi$)

(ii) Whole numbers less than 5 are 0, 1, 2, 3, 4.

Given $2n$ i.e., $2 \times 0, 2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4$ i.e., 0, 2, 4, 6, 8.

The given set i.e., A can be written as $\{0, 2, 4, 6, 8\}$

(Every set is a subset of universal set i.e., $A \subset \xi$)

(iii) Integers less than 5 are, -4, -3, -2, -1, 0, 1, 2, 3, 4

Given $2n$ i.e., $2 \times -2, 2 \times -1, 2 \times 0, 2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4$,

i.e.-4, -2, 0, 2, 4, 6, 8.

The given set i.e. A can be written as $\{..., -4, -2, 0, 2, 4, 6, 8\}$

(Every set is a subset of universal set i.e., $A \subset \xi$)