

Exercise 4

1. Solve the inequation, $3x - 11 < 3$ where $x \in \{1, 2, 3, \dots, 10\}$. Also, represent its solution on a number line.

Solution:

Given inequation, $3x - 11 < 3$

$$3x < 3 + 11$$

$$3x < 14$$

$$\Rightarrow x < 14/3$$

But, $x \in \{1, 2, 3, \dots, 10\}$

Hence, the solution set is $\{1, 2, 3, 4\}$.

Representing the solution on a number line:

2. Solve $2(x - 3) < 1$, $x \in \{1, 2, 3, \dots, 10\}$

Solution:

Given inequation, $2(x - 3) < 1$

$$2x - 6 < 1$$

$$2x < 7$$

$$\Rightarrow x < 7/2$$

But, $x \in \{1, 2, 3, \dots, 10\}$

Hence, the solution set is $\{1, 2, 3\}$

3. Solve $5 - 4x > 2 - 3x$, $x \in \mathbb{W}$. Also represent its solution on the number line.

Solution:

Given inequation, $5 - 4x > 2 - 3x$

$$-4x + 3x > 2 - 5$$

$$-x > -3$$

On multiplying both sides by -1 , the inequality reverses

$$\Rightarrow x < 3$$

Since, $x \in \mathbb{W}$

The solution set is $\{0, 1, 2\}$

Representing the solution on a number line:

4. List the solution set of $30 - 4(2x - 1) < 30$, given that x is a positive integer.

Solution:

Given inequation, $30 - 4(2x - 1) < 30$

$$30 - 8x + 4 < 30$$

$$34 - 8x < 30$$

$$-8x < 30 - 34$$

$$-8x < -4 \text{ [On multiplying both sides by } -1, \text{ the inequality reverses]}$$

$$8x > 4$$

$$x > 4/8$$

$$\Rightarrow x > 1/2$$

As x is a positive integer

The solution set is $\{1, 2, 3, \dots\}$

5. Solve: $2(x - 2) < 3x - 2$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

Solution:

Given inequation, $2(x - 2) < 3x - 2$

$$2x - 4 < 3x - 2$$

$$2x - 3x < -2 + 4$$

$$-x < 2$$

$$\Rightarrow x > -2$$

But, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Hence, the solution set is $\{-1, 0, 1, 2, 3\}$.

6. If x is a negative integer, find the solution set of $2/3 + 1/3(x + 1) > 0$.

Solution:

Given inequation, $2/3 + 1/3(x + 1) > 0$.

$$2/3 + x/3 + 1/3 > 0$$

$$x/3 + 1 > 0$$

$$x/3 > -1$$

$$\Rightarrow x > -3$$

As x is a negative integer

The solution set is $\{-1, -2\}$.

7. Solve: $(2x - 3)/4 \geq 1/2$, $x \in \{0, 1, 2, \dots, 8\}$

Solution:

Given inequation, $(2x - 3)/4 \geq 1/2$

$$2x - 3 \geq 4 \times 1/2$$

$$2x - 3 \geq 2$$

$$2x \geq 2 + 3$$

$$2x \geq 5$$

$$\Rightarrow x \geq 5/2$$

But, $x \in \{0, 1, 2, \dots, 8\}$

Hence, the solution set is $\{3, 4, 5, 6, 7, 8\}$.

8. Solve $x - 3(2 + x) > 2(3x - 1)$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$. Also represent its solution on the number line.

Solution:

Given inequation, $x - 3(2 + x) > 2(3x - 1)$

$$x - 6 - 3x > 6x - 2$$

$$-2x - 6 > 6x - 2$$

$$-6x - 2x > -2 + 6$$

$$-8x > 4$$

$$x < -4/8$$

$$\Rightarrow x < -1/2$$

But, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Hence, the solution set is $\{-3, -2, -1\}$

9. Given $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$ solve $x - 3 < 2x - 1$.

Solution:

Given inequation, $x - 3 < 2x - 1$

$$x - 2x < -1 + 3$$

$$-x < 2$$

$$\Rightarrow x > -2$$

But, $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$

Hence, the solution set is $\{1, 2, 3, 4, 5, 6, 7, 9\}$.

10. Given $A = \{x: x \in \mathbb{I}, -4 \leq x \leq 4\}$, solve $2x - 3 < 3$ where x has the domain A .

Graph the solution

set on the number line.

Solution:

Given equation, $2x - 3 < 3$

$$2x < 6$$

$$\Rightarrow x < 3$$

But x has a the domain $A = \{x: x \in \mathbb{I}, -4 \leq x \leq 4\}$

$$A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

Hence, the solution set is $\{-4, -3, -2, -1, 0, 1, 2\}$.

Representing the solution on a number line:

11. List the solution set of the inequation

$$\frac{1}{2} + 8x > 5x - \frac{3}{2}, x \in \mathbb{Z}$$

Solution:

Given inequation, $\frac{1}{2} + 8x > 5x - \frac{3}{2}$

$$8x - 5x > -\frac{3}{2} - \frac{1}{2}$$

$$3x > -\frac{4}{2}$$

$$\Rightarrow x > -\frac{2}{3}$$

As $x \in \mathbb{Z}$

The solution set is $\{0, 1, 2, 3, 4, 5, \dots\}$

12. List the solution set of $(11 - 2x)/5 \geq (9 - 3x)/8 + 3/4, x \in \mathbb{N}$

Solution:

Given inequation, $(11 - 2x)/5 \geq (9 - 3x)/8 + 3/4$

$$(11 - 2x)/5 \geq (9 - 3x + 6)/8$$

$$8(11 - 2x) \geq 5(15 - 3x)$$

$$88 - 16x \geq 75 - 15x$$

$$15x - 16x \geq 75 - 88$$

$$-x \geq -13$$

$$\Rightarrow x \leq 13$$

As $x \in \mathbb{N}$

Hence, the solution set is $\{1, 2, 3, 4, \dots, 13\}$.

13. Find the values of x , which satisfy the inequation : , $x \in \mathbb{N}$. Graph the solution set on the number line.

Solution:

Given inequation,

$$-2 \leq (3 - 4x) / 6 \leq 11/6$$

$$-12 \leq 3 - 4x \leq 11$$

$$-12 - 3 \leq -4x \leq 11 - 3$$

$$-15 \leq -4x \leq 8$$

$$-15/4 \leq -x \leq 8/4$$

$$\Rightarrow 15/4 \geq x \geq -2$$

As $x \in \mathbb{N}$,

The solution set is $\{1, 2, 3\}$.

Representing the solution on a number line:

14. If $x \in \mathbb{W}$, find the solution set of $3/5 x - (2x - 1)/3 > 1$. Also graph the solution set on the number line, if possible.

Solution:

Given inequation, $3/5 x - (2x - 1)/3 > 1$

$$9/15 x - 5(2x - 1)/15 > 1 \text{ [Taking L.C.M]}$$

$$9x - 5(2x - 1) > 15 \text{ [Multiplying by 15 on both sides]}$$

$$9x - 10x + 5 > 15$$

$$-x > 15 - 5$$

$$-x > 10$$

$$\Rightarrow x < -10$$

But, $x \in \mathbb{W}$

Hence, the solution set is a null set.

Thus, it can't be represented on number line.

15. Solve:

(i) $x/2 + 5 \leq x/3 + 6$ where x is a positive odd integer.

(ii) $(2x + 3)/3 \geq (3x - 1)/4$ where x is positive even integer.

Solution:

(i) Given inequation, $x/2 + 5 \leq x/3 + 6$

$$(x + 10)/2 \leq (x + 18)/3 \text{ [Taking L.C.M on both sides]}$$

$$3(x + 10) \leq 2(x + 18) \text{ [On cross-multiplying]}$$

$$3x + 30 \leq 2x + 36$$

$$3x - 2x \leq 36 - 30$$

$$\Rightarrow x \leq 6$$

As x is a positive odd integer.

Hence, the solution set is $\{1, 3, 5\}$.

(ii) Given inequation, $(2x + 3)/3 \geq (3x - 1)/4$

$4(2x + 3) \geq 3(3x - 1)$ [On cross-multiplying]

$$8x + 12 \geq 9x - 3$$

$$-9x + 8x \geq -12 - 3$$

$$-x \geq -15$$

$$\Rightarrow x \leq 15$$

As x is positive even integer.

Hence, the solution set is $\{2, 4, 6, 8, 10, 12, 14\}$.

16. Given that $x \in \mathbf{I}$, solve the inequation and graph the solution on the number line:

$$3 \geq (x - 4)/2 + x/3 \geq 2$$

Solution:

Given inequation, $3 \geq (x - 4)/2 + x/3 \geq 2$

Now, let's take

$3 \geq (x - 4)/2 + x/3$, we have

$$3 \geq (3x - 12 + 2x)/6 \text{ [Taking L.C.M]}$$

$$18 \geq 5x - 12$$

$$30 \geq 5x$$

$$\Rightarrow x \leq 6 \dots (i)$$

Next,

$$(x - 4)/2 + x/3 \geq 2$$

$$(3x - 12 + 2x)/6 \geq 2$$

$$5x - 12 \geq 12$$

$$5x \geq 24$$

$$x \geq 24/5 \Rightarrow x \geq 4.8 \dots (ii)$$

Hence, from (i) and (ii) we have

Solution of $x = \{5, 6\}$

Representing the solution on a number line:

17. Given $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$, find the values of x for which $-3 < 2x - 1 < x + 4$.

Solution:

Given inequation, $-3 < 2x - 1 < x + 4$

So, we have

$$-3 < 2x - 1 \text{ and } 2x - 1 < x + 4$$

$$-2x < 3 - 1 \text{ and } 2x - x < 4 + 1$$

$$-2x < 2 \text{ and } x < 5$$

$$x > -1 \text{ and } x < 5$$

As $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$

The solution set is $\{1, 2, 3, 4\}$.

18. Solve: $1 \geq 15 - 7x > 2x - 27$, $x \in \mathbf{N}$

Solution:

Given inequation, $1 \geq 15 - 7x > 2x - 27$,

So, we have

$$1 \geq 15 - 7x \text{ and } 15 - 7x > 2x - 27$$

$$7x \geq 15 - 1 \text{ and } -2x - 7x > -27 - 15$$

$$7x \geq 14 \text{ and } -9x > -42$$

$$x \geq 2 \text{ and } -x > -42/9$$

$$x \geq 2 \text{ and } x < 14/3$$

$$\Rightarrow 2 \leq x < 14/3$$

But as $x \in \mathbb{N}$

The solution set is $\{2, 3, 4\}$.

19. If $x \in \mathbb{Z}$, solve $2 + 4x < 2x - 5 \leq 3x$. Also represent its solution on the number line.

Solution

Given inequation, $2 + 4x < 2x - 5 \leq 3x$

So, we have

$$2 + 4x < 2x - 5 \text{ and } 2x - 5 \leq 3x$$

$$4x - 2x < -5 - 2 \text{ and } 2x - 3x \leq 5$$

$$2x < -7 \text{ and } -x \leq 5$$

$$x < -7/2 \text{ and } x \geq -5$$

$$\Rightarrow -5 \leq x < -7/2$$

As $x \in \mathbb{Z}$

The solution set is $\{-5, -4\}$.

Representing the solution on a number line:

20. Solve the inequation $12 + \leq 5 + 3x$, $x \in \mathbb{R}$. Represent the solution on a number line.

Solution:

Given inequation, $12 + \leq 5 + 3x$

$$12 + 11/6 x \leq 5 + 3x$$

$$72 + 11x \leq 30 + 18x \text{ [Multiplying by 6 on both sides]}$$

$$11x - 18x \leq 30 - 72$$

$$-7x \leq -42$$

$$-x \leq -6$$

$$x \geq 6$$

As $x \in \mathbb{R}$

The solution set is $\{x : x \in \mathbb{R}, x \geq 6\}$

Representing the solution on a number line:

21. Solve: $(4x - 10)/3 \leq (5x - 7)/2$, $x \in \mathbb{R}$ and represent the solution set on the number line.

Solution:

Given inequation, $(4x - 10)/3 \leq (5x - 7)/2$

$$2(4x - 10) \leq 3(5x - 7) \text{ [On cross-multiplying]}$$

$$8x - 20 \leq 15x - 21$$

$$8x - 15x \leq -21 + 20$$

$$-7x \leq -1$$

$$-x \leq -1/7$$

$$x \geq 1/7$$

As $x \in \mathbb{R}$

Hence, the solution set is $\{x: x \in \mathbb{R}, x \geq 1/7\}$

Representing the solution on a number line:

22. Solve $3x/5 - (2x - 1)/3 > 1$, $x \in \mathbb{R}$ and represent the solution set on the number line.

Solution:

Given inequation, $3x/5 - (2x - 1)/3 > 1$

$(9x - 10x + 5)/15 > 1$ [Taking L.C.M]

$$-x + 5 > 15$$

$$-x > 15 - 5$$

$$-x > 10$$

$$x < -10$$

As $x \in \mathbb{R}$

Hence, the solution set is $\{x: x \in \mathbb{R}, x < -10\}$

Representing the solution on a number line:

23. Solve the inequation $-3 \leq 3 - 2x < 9$, $x \in \mathbb{R}$. Represent your solution on a number line.

Solution:

Given inequation, $-3 \leq 3 - 2x < 9$

$$-3 - 3 \leq -2x < 9 - 3$$

$$-6 \leq -2x < 6$$

$$-3 \leq -x < 3$$

$$-3 < x \leq 3$$

As $x \in \mathbb{R}$

The solution set is $\{x: x \in \mathbb{R}, -3 < x \leq 3\}$

Representing the solution on a number line:

24. Solve $2 \leq 2x - 3 \leq 5$, $x \in \mathbb{R}$ and mark it on a number line.

Solution:

Given inequation, $2 \leq 2x - 3 \leq 5$

$$2 + 3 \leq 2x \leq 5 + 3$$

$$5 \leq 2x \leq 8$$

$$5/2 \leq x \leq 4$$

Hence, the solution set is $\{x: x \in \mathbb{R}, 5/2 \leq x \leq 4\}$

Representing the solution on a number line:

25. Given that $x \in \mathbb{R}$, solve the following inequation and graph the solution on the number line: -1

$$\leq 3 + 4x < 23.$$

Solution:

Given inequation, $-1 \leq 3 + 4x < 23$

$$-1 - 3 \leq 4x < 23 - 3$$

$$-4 \leq 4x < 20$$

$$-4/4 \leq x < 20/4$$

$$-1 \leq x < 5$$

Hence, the solution set is $\{-1 \leq x < 5; x \in \mathbb{R}\}$

Representing the solution on a number line:

26. Solve the following inequation and graph the solution on the number line.

, $x \in \mathbb{R}$

Solution:

Given inequation,

$$-8/3 \leq (3x + 1)/3 < 10/3$$

$$-8 \leq 3x + 1 < 10 \text{ [Multiplying by 3]}$$

$$-8 - 1 \leq 3x < 10 - 1$$

$$-9 \leq 3x < 9$$

$$-3 \leq x < 3 \text{ [Dividing by 3]}$$

Thus, the solution set is $\{x: x \in \mathbb{R}, -3 \leq x < 3\}$

Representing the solution on a number line:

27. Solve the following inequation and represent the solution set on the number line:

Solution:

Given in equation,

$$-3 < -(3 + 4x)/6 \leq 5/6 \text{ [Taking L.C.M]}$$

$$-18 < -3 - 4x \leq 5 \text{ [Multiplying by 6]}$$

$$-18 + 3 < -4x \leq 5 + 3$$

$$-15 < -4x \leq 8$$

$$-15/4 < -x \leq 8/4$$

$$-2 \leq x < 15/4$$

Hence, the solution set is $\{x: x \in \mathbb{R}, -2 \leq x < 15/4\}$

Representing the solution on a number line:

28. Solve . Also graph the solution set on the number line

Solution:

Given inequation,

$$[2x + 1 + 4(3 - x)]/2 \geq 7 \text{ [Taking L.C.M]}$$

$$2x + 1 + 12 - 4x \geq 14$$

$$-2x + 13 \geq 14$$

$$-2x \geq 14 - 13$$

$$-2x \geq 1$$

$$-x \geq \frac{1}{2}$$

$$x \leq -\frac{1}{2}$$

Hence, the solution set is $\{x : x \in \mathbb{R}, x \leq -\frac{1}{2}\}$

Representing the solution on a number line:

29. Solving the following inequation, write the solution set and represent it on the number line -

$$3(x - 7) \geq 15 - 7x, x \in \mathbb{R}$$

Solution:

$$\text{Given inequation, } -3(x - 7) \geq 15 - 7x >$$

$$-3x + 21 \geq 15 - 7x > (x + 1)/3$$

So,

$$-3x + 21 \geq 15 - 7x$$

$$7x - 3x \geq 15 - 21$$

$$4x \geq -6$$

$$x \geq -\frac{6}{4}$$

$$x \geq -\frac{3}{2}$$

And,

$$15 - 7x > (x + 1)/3$$

$$3(15 - 7x) > x + 1$$

$$45 - 21x > x + 1$$

$$-21x - x > 1 - 45$$

$$-22x > -44$$

$$-x > -44/22$$

$$x < 2$$

Hence, the solution set is $\{x : x \in \mathbb{R}, -\frac{3}{2} \leq x < 2\}$

Representing the solution on a number line:

30. Solve the inequation :

. Graph the solution set on the number line.

Solution:

Given inequation,

$$-\frac{5}{2} + 2x \leq \frac{4x}{3} \leq \frac{4}{3} + 2x$$

So, we have

$$-\frac{5}{2} + 2x \leq \frac{4x}{3} \text{ and } \frac{4x}{3} \leq \frac{4}{3} + 2x$$

$$2x - \frac{4x}{3} \leq \frac{5}{2} \text{ and } \frac{4x}{3} - 2x \leq \frac{4}{3}$$

$$\frac{(6x - 4x)}{3} \leq \frac{5}{2} \text{ and } \frac{(4x - 6x)}{3} \leq \frac{4}{3} \text{ [Taking L.C.M]}$$

$$2x \leq \frac{15}{2} \text{ and } -2x \leq 4$$

$$x \leq \frac{15}{4} \text{ and } -x \leq 2 \text{ [Dividing by 2]}$$

$$x \leq \frac{15}{4} \text{ and } x \geq -2$$

$$-2 \leq x \leq \frac{15}{4}$$

For $x \in \mathbb{W}$

Thus, the solution set is $\{0, 1, 2, 3\}$

Representing the solution on a number line:

31. Solve the inequality $2x - 5 \leq 5x + 4 < 11$, where $x \in \mathbb{I}$. Also represent the solution set on the number line.

Solution:

Given inequality, $2x - 5 \leq 5x + 4 < 11$

So, we have

$$2x - 5 \leq 5x + 4 \text{ and } 5x + 4 < 11$$

$$2x - 5x \leq 4 + 5 \text{ and } 5x < 11 - 4$$

$$-3x \leq 9 \text{ and } 5x < 7$$

$$-x \leq 9/3 \text{ and } x < 7/5$$

$$x \geq -3 \text{ and } x < 7/5$$

$$-3 \leq x < 7/5$$

As $x \in \mathbb{I}$

Thus, the solutions set is $\{-3, -2, -1, 0, 1\}$

Representing the solution on a number line:

32. If $x \in \mathbb{I}$, A is the solution set of $2(x - 1) < 3x - 1$ and B is the solution set of $4x - 3 \leq 8 + x$, find

$A \cap B$.

Solution:

Given inequations,

$$2(x - 1) < 3x - 1 \text{ and } 4x - 3 \leq 8 + x \text{ for } x \in \mathbb{I}$$

Solving for both, we have

$$2x - 3x < 2 - 1 \text{ and } 4x - x \leq 8 + 3$$

$$-x < 1 \text{ and } 3x \leq 11$$

$$x > -1 \text{ and } x \leq 11/3$$

Hence,

$$\text{Solution set } A = \{0, 1, 2, 3, \dots\}$$

$$\text{Solution set } B = \{3, 2, 1, 0, -1, \dots\}$$

$$\text{Thus, } A \cap B = \{0, 1, 2, 3\}$$

33. If P is the solution set of $-3x + 4 < 2x - 3$, $x \in \mathbb{N}$ and Q is the solution set of $4x - 5 < 12$, $x \in \mathbb{W}$,

find

(i) $P \cap Q$

(ii) $Q - P$.

Solution:

Given inequations,

$$-3x + 4 < 2x - 3 \text{ where } x \in \mathbb{N} \text{ and } 4x - 5 < 12 \text{ where } x \in \mathbb{W}$$

So, solving

$$-3x + 4 < 2x - 3 \text{ where } x \in \mathbb{N}$$

$$-3x - 2x < -3 - 4$$

$$-5x < -7$$

$$x > 7/5$$

Hence, the solution set P is $\{2, 3, 4, 5, \dots\}$

And, solving

$$4x - 5 < 12 \text{ where } x \in W$$

$$4x < 12 + 5$$

$$4x < 17$$

$$x < 17/4$$

Hence, the solution set Q is $\{0, 1, 2, 3, 4\}$

Therefore,

$$(i) P \cap Q = \{2, 3, 4\}$$

$$(ii) Q - P = \{0, 1\}$$

34. A = $\{x : 11x - 5 > 7x + 3, x \in R\}$ and B = $\{x : 18x - 9 \geq 15 + 12x, x \in R\}$

Find the range of set $A \cap B$ and represent it on a number line

Solution:

Given, A = $\{x : 11x - 5 > 7x + 3, x \in R\}$ and B = $\{x : 18x - 9 \geq 15 + 12x, x \in R\}$

Solving for A,

$$11x - 5 > 7x + 3$$

$$11x - 7x > 3 + 5$$

$$4x > 8$$

$$x > 2$$

Hence, A = $\{x : x > 2, x \in R\}$

Next, solving for B

$$18x - 9 \geq 15 + 12x$$

$$18x - 12x \geq 15 + 9$$

$$6x \geq 24$$

$$x \geq 4$$

Hence, B = $\{x : x \geq 4, x \in R\}$

Thus, $A \cap B = x \geq 4$

Representing the solution on a number line:

35. Given: P $\{x : 5 < 2x - 1 \leq 11, x \in R\}$

Q $\{x : -1 \leq 3 + 4x < 23, x \in I\}$ where

R = (real numbers), I = (integers)

Represent P and Q on number line. Write down the elements of $P \cap Q$.

Solution:

Given, P $\{x : 5 < 2x - 1 \leq 11, x \in R\}$ and Q $\{x : -1 \leq 3 + 4x < 23, x \in I\}$

Solving for P,

$$5 < 2x - 1 \leq 11$$

$$5 + 1 < 2x \leq 11 + 1$$

$$6 < 2x \leq 12$$

$$3 < x \leq 6$$

Hence, $P = \{x : 3 < x \leq 6, x \in \mathbb{R}\}$

Representing the solution on a number line:

Next, solving for Q

$$-1 \leq 3 + 4x < 23$$

$$-1 - 3 \leq 4x < 23 - 3$$

$$-4 \leq 4x < 20$$

$$-1 \leq x < 5$$

Hence, solution $Q = \{-1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:

Therefore, $P \cap Q = \{4\}$

36. If $x \in \mathbb{I}$, find the smallest value of x which satisfies the inequation

Solution:

Given inequation,

$$(4x + 5)/2 > (5x + 6)/3 \text{ [Taking L.C.M]}$$

$$3(4x + 5) > 2(5x + 6) \text{ [On cross-multiplication]}$$

$$12x + 15 > 10x + 12$$

$$12x - 10x > 12 - 15$$

$$2x > -3$$

$$x > -3/2$$

Hence, for $x \in \mathbb{I}$ the smallest value of x is -1.

37. Given $20 - 5x < 5(x + 8)$, find the smallest value of x, when

(i) $x \in \mathbb{I}$

(ii) $x \in \mathbb{W}$

(iii) $x \in \mathbb{N}$.

Solution:

Given inequation, $20 - 5x < 5(x + 8)$

$$20 - 5x < 5x + 40$$

$$-5x - 5x < 40 - 20$$

$$-10x < 20$$

$$-x < 20/10$$

$$x > -2$$

Thus,

(i) For $x \in \mathbb{I}$, the smallest value = -1

(ii) For $x \in \mathbb{W}$, the smallest value = 0

(iii) For $x \in \mathbb{N}$, the smallest value = 1

38. Solve the following inequation and represent the solution set on the number line:

Solution:

Given inequation,

So, we have

$$4x - 19 < 3x/5 - 2 \text{ and } 3x/5 - 2 \leq -2/5 + x$$

$$4x - 3x/5 < 19 - 2 \text{ and } 3x/5 - x \leq 2 - 2/5$$

$$(20x - 3x)/5 < 17 \text{ and } (3x - 5x)/5 \leq (10 - 2)/5$$

$$17x < 35 \text{ and } -2x \leq 8 \text{ [Multiplying by 5]}$$

$$x < 5 \text{ and } -x \leq 4$$

$$x < 5 \text{ and } x \geq 4$$

$$-4 \leq x < 5, x \in \mathbb{R}$$

Hence, the solution set is $\{x : -4 \leq x < 5, x \in \mathbb{R}\}$

Representing the solution on a number line:

39. Solve the given inequation and graph the solution on the number line:

$$2y - 3 < y + 1 \leq 4y + 7; y \in \mathbb{R}.$$

Solution:

Given inequation, $2y - 3 < y + 1 \leq 4y + 7$

So, we have

$$2y - 3 < y + 1 \text{ and } y + 1 \leq 4y + 7$$

$$2y - y < 1 + 3 \text{ and } y - 4y \leq 7 - 1$$

$$y < 4 \text{ and } -3y \leq 6$$

$$y < 4 \text{ and } -y \leq 2 \Rightarrow y \geq -2$$

$$\text{Thus, } -2 \leq y < 4$$

The solution set is $\{y : -2 \leq y < 4, y \in \mathbb{R}\}$

Representing the solution on a number line:

40. Solve the inequation and represent the solution set on the number line.

Solution:

Given inequation,

So, we have

$$-3 + x \leq 8x/3 + 2 \text{ and } 8x/3 + 2 \leq 14/3 + 2x$$

$$x - 8x/3 \leq 2 + 3 \text{ and } 8x/3 - 2x \leq 14/3 - 2$$

$$(3x - 8x)/3 \leq 5 \text{ and } (8x - 6x)/3 \leq (14 - 6)/3 \text{ [Taking L.C.M]}$$

$$-5x/3 \leq 5 \text{ and } 2x \leq 8$$

$$-5x \leq 15 \text{ and } x \leq 8/2$$

$$-x \leq 3 \text{ and } x \leq 4$$

$$x \geq -3 \text{ and } x \leq 4$$

$$\Rightarrow -3 \leq x \leq 4$$

Thus, the solution set is $\{-3, -2, -1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:

41. Find the greatest integer which is such that if 7 is added to its double, the resulting number

becomes greater than three times the integer.

Solution:

Let's consider the greatest integer to be x

Then according to the given condition, we have

$$2x + 7 > 3x$$

$$2x - 3x > -7$$

$$-x > -7$$

$$x < 7, x \in \mathbb{R}$$

Hence, the greatest integer value is 6.

42. One-third of a bamboo pole is buried in mud, one-sixth of it is in water and the part above the water is greater than or equal to 3 metres. Find the length of the shortest pole.

Solution:

Let's assume the length of the shortest pole = x metre

Now,

Length of the pole which is buried in mud = $x/3$

Length of the pole which is in the water = $x/6$

Then according to the given condition, we have

$$x - [x/3 + x/6] \geq 3$$

$$x - [(2x + x)/6] \geq 3$$

$$x - 3x/6 \geq 3$$

$$x - x/2 \geq 3$$

$$x/2 \geq 3$$

$$x \geq 6 \text{ [Multiplying by 6]}$$

Therefore, the length of the shortest pole is 6 metres.

Chapter Test

1. Solve the inequation: $5x - 2 \leq 3(3 - x)$ where $x \in \{-2, -1, 0, 1, 2, 3, 4\}$. Also represent its solution on the number line.

Solution:

Given inequation, $5x - 2 \leq 3(3 - x)$

$$5x - 2 \leq 9 - 3x$$

$$5x + 3x \leq 9 + 2$$

$$8x \leq 11$$

$$x \leq 11/8$$

As $x \in \{-2, -1, 0, 1, 2, 3, 4\}$

The solution set is $\{-2, -1, 0, 1\}$

Representing the solution on a number line:

2. Solve the inequation: $6x - 5 < 3x + 4$, $x \in \mathbb{I}$

Solution:

Given inequation, $6x - 5 < 3x + 4$

$$6x - 3x < 4 + 5$$

$$3x < 9$$

$$x < 9/3$$

$$x < 3$$

As $x \in \mathbb{I}$

The solution set is $\{2, 1, 0, -1, -2, \dots\}$

3. Find the solution set of the inequation $x + 5 \leq 2x + 3$; $x \in \mathbb{R}$

Graph the solution set on the number line.

Solution:

Given inequation, $x + 5 \leq 2x + 3$

$$x - 2x \leq 3 - 5$$

$$-x \leq -2$$

$$x \geq 2$$

As $x \in \mathbb{R}$

Thus, the solution set is $\{2, 3, 4, 5, \dots\}$

Representing the solution on a number line:

4. If $x \in \mathbb{R}$ (real numbers) and $-1 < 3 - 2x \leq 7$, find solution set and present it on a number line.

Solution:

Given inequation, $-1 < 3 - 2x \leq 7$

$$-1 - 3 < -2x \leq 7 - 3$$

$$-4 < -2x \leq 4$$

$$-4/2 < -x \leq 4/2$$

$$-2 < -x \leq 2$$

Thus, $-2 \leq x < 2$

The solution set is $\{x : x \in \mathbb{R}, -2 \leq x < 2\}$

Representing the solution on a number line:

5. Solve the inequation:

Solution:

Given inequation,

$$(5x + 1)/7 - 4(5x + 14)/35 \leq 8/5 + (3x - 1)/7$$

$$[5(5x + 1) - 4(5x + 14)]/35 \leq [56 + 5(3x - 1)]/35 \text{ [Taking L.C.M]}$$

$$(25x + 5 - 20x - 56) \leq 56 + 15x - 5$$

$$5x - 51 \leq 51 + 15x$$

$$5x - 15x \leq 51 + 51$$

$$-10x \leq 102$$

$$-x \leq 102/10$$

$$x \geq -51/5$$

Hence, the solution set is $\{x : x \in \mathbb{R}, x \geq -51/5\}$