Chapter 5 Quadratic Equations in One Variable Ex 5.4

Question 1.
Find the discriminant of the following equations and hence find the nature of roots:
(i) $3x^2 - 5x - 2 = 0$
(ii) $2x^2 - 3x + 5 = 0$
(iii) $7x^2 + 8x + 2 = 0$
(iv) $3x^2 + 2x - 1 = 0$
(v) $16x^2 - 40x + 25 = 0$
(vi) $2x^2 + 15x + 30 = 0.$
Solution:

(i) $3x^2 - 5x - 2 = 0$

Here a = 3, b = -5, c = -2

$\therefore D = b^2 - 4ac = (-5)^2 - 4 \times 3 \times (-2) = 25 + 24 = 49$

$\therefore$ Discriminant $= 49$

$\because$ $D > 0$

$\therefore$ Roots are real and distinct

(ii) $2x^2 - 3x + 5 = 0$

Here $a = 2, b = -3, c = 5$

$\therefore$ $D = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31$

$\therefore$ Discriminant $= -31$

$\because$ $D < 0$,

$\therefore$ Roots are not real.

(iii) $7x^2 + 8x + 2 = 0$

Here $a = 7, b = 8, c = 2$

$\therefore$ $D = b^2 - 4ac = (8)^2 - 4 \times 7 \times 2 = 64 - 56 = 8$

$\therefore$ Discriminant $= 8$

$\because$ $D > 0$

$\therefore$ Roots are real and distinct

(iv) $3x^2 + 2x - 1 = 0$

Here $a = 3, b = 2, c = -1$

$\therefore$ $D = b^2 - 4ac = (2)^2 - 4 \times 3 \times (-1) = 4 + 12 = 16$

$\therefore$ Discriminant $= 16$

$\because$ $D > 0$

$\therefore$ Roots are real and distinct

(v) $16x^2 - 40x + 25 = 0$

$a = 16, b = -40, c = 25$

$\therefore D = b^2 - 4ac = (-40)^2 - 4 \times 16 \times 25 = 1600 - 1600 = 0$

$\therefore$ Discriminant $= 0$

$\because$ $D = 0$

$\therefore$ Roots are real and equal.

(vi) $2x^2 + 15x + 30 = 0$

Here $a = 2, b = 15, c = 30$

$\therefore D = b^2 - 4ac = (15)^2 - 4 \times 2 \times 30 = 225 - 240 = -15$

$\therefore$ Discriminant $= -15$

$\because$ $D < 0$

$\therefore$ Root are not real.

Question 2.
Discuss the nature of the roots of the following quadratic equations:

(i) $x^2 - 4x - 1 = 0$

(ii) $3x^2 - 2x + 13 = 0$

(iii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iv) $x^2 - 12x + 4 = 0$

(v) $-2x^2 + x + 1 = 0$

(vi) $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Solution:

(i) $x^2 - 4x - 1 = 0$

Here a = 1, b = -4, c = -1

$\therefore$ $D = b^2 - 4ac = (-4)^2 - 4 \times 1 \times (-1) = 16 + 4 = 20$

$\because$ $D > 0$

Roots are real and distinct

(ii) $3x^2 - 2x + \frac{1}{3} = 0$

Here $a = 3, b = -2, c = \frac{1}{3}$

$\therefore$ $D = b^2 - 4ac = (-2)^2 - 4 \times 3 \times \frac{1}{3} = 4 - 4 = 0$

$\because$ $D = 0$

$\therefore$ Roots are real and equal.

(iii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Here $a = 3$, $b = -4\sqrt{3}$, $c = 4$

$\therefore D = b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4 = 48 - 48 = 0$

$\because$ $D = 0$

$\therefore$ Roots are real and equal

(iv) $x^2 - \frac{1}{2}x + 4 = 0$

Here $a = 1$, $b = -\frac{1}{2}$, $c = 4$

$\therefore D = b^2 - 4ac = \left(-\frac{1}{2}\right) - 4 \times 1 \times 4 = \frac{1}{4} - 16 = -\frac{63}{4}$

$\because$ D < 0

$\therefore$ Roots are not real.

(v) $-2x^2 + x + 1 = 0$

Here, $a = -2$, $b = 1$, $c = 1$

$D = b^2 - 4ac = (1)^2 - 4 \times (-2) \times 1 = 1 + 8 = 9$

$\because$ D > 0

$\therefore$ Roots are real and distinct.

(vi) $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Here $a = 2\sqrt{3}$, $b = -5$, $c = \sqrt{3}$

$\therefore D = b^2 - 4ac = (-5)^2 - 4 \times 2\sqrt{3} \times \sqrt{3} = 25 - 24 = 1$

$\because$ D > 0

$\therefore$ Roots are real and distinct.

Question 3.
Find the nature of the roots of the following quadratic equations:
(i) $x^2 - 12X - 12 = 0$
(ii) $x^2 - 2\sqrt{3}x - 1 = 0$ If real roots exist, find them.

Solution:

(i) $x^2 - \frac{1}{2}x - \frac{1}{2} = 0$

Here a = 1, b = $-\frac{1}{2}$, c = $-\frac{1}{2}$

$\therefore$ D = $b^2 - 4ac$

$= \left(\frac{-1}{2}\right)^2 - 4 \times 1 \times \left(\frac{-1}{2}\right) = \frac{1}{4} + 2 = \frac{9}{4}$

$\because D = \frac{9}{4} > 0$

$\therefore$ Roots are real and unequal

(ii) $x^2 - 2\sqrt{3}x - 1 = 0$

Here $a = 1$, $b = -2\sqrt{3}$, $c = -1$

$\therefore D = b^2 - 4ac$

$= \left(-2\sqrt{3}\right)^2 - 4 \times 1 \times (-1) = 12 + 4 = 16$

$\because D > 0$

$\therefore$ Roots are real and unequal.

Question 4.

Without solving the following quadratic equation, find the value of 'p' for which the given equations have real and equal roots:

(i) $px^2 - 4x + 3 = 0$

(ii) $x^2 + (p - 2)x + p = 0$.

Solution:

(i) $px^2 - 4x + 3 = 0$

Here a = p, b = -4, c = 3

$\therefore$ $D = b^2 - 4ac = (-4)^2 - 4 \times p \times 3 = 16 - 12p$

$\because$ The roots are equal

$\therefore$ $D = 0$

$\Rightarrow b^2 - 4ac = 0 \Rightarrow 16 - 12p = 0 \Rightarrow 12p = 16$

$\Rightarrow p = \frac{16}{12} = \frac{4}{3}$ $\quad\therefore\quad p = \frac{4}{3}$

(ii) $x^2 + (p - 3)x + p = 0$

Here $a = 1$, $b = (p - 3)$, $c = p$

$\because$ Equation has real and equal roots

$\therefore$ $b^2 - 4ac = 0$

$\Rightarrow (p-3)^2 - 4(1)(p) = 0 \Rightarrow (p-3)^2 - 4p = 0$

$\Rightarrow p^2 + 9 - 6p - 4p = 0 \Rightarrow p^2 - 10p + 9 = 0$

$\Rightarrow p^2 - 9p - p + 9 = 0 \Rightarrow p(p-9) - 1(p-9) = 0$

$\Rightarrow (p-1)(p-9) = 0 \quad \therefore p = 1, 9$

Question 5.
Find the value (s) of k for which each of the following quadratic equation has equal roots:
(i) $kx^2 - 4x - 5 = 0$
(ii) $(k - 4)x^2 + 2(k - 4)x + 4 = 0$
Solution:

(i) $kx^2 - 4x - 5 = 0$

Here a = k, b = -4, c = 5

$\therefore D = b^2 - 4ac = (-4)^2 - 4 \times k \times (-5) = 16 + 20k$

$\because$ Roots are equal.

$\therefore D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\therefore 16 + 20k = 0 \Rightarrow 20k = -16$

$\Rightarrow k = \frac{-16}{20} = \frac{-4}{5}$

Hence $k = \frac{-4}{5}$

(ii) $(k - 4)x^2 + 2(k - 4)x + 4 = 0$

Here $a = k - 4$, $b = 2(k - 4)$, $c = 4$

$D = b^2 - 4ac$

$= [2(k - 4)]^2 - 4 \times (k - 4) \times 4$

$= 4(k^2 + 16 - 8k) - 16(k - 4)$

$= 4(k^2 - 8k + 16) - 16(k - 4)$

$= 4[k^2 - 8k + 16 - 4k + 16]$

$= 4(k^2 - 12k + 32)$

$\because$ Roots are equal

$\therefore D = 0$

$\Rightarrow 4(k^2 - 12k + 32) = 0$

$\Rightarrow k^2 - 12k + 32 = 0$

$\Rightarrow k^2 - 8k - 4k + 32 = 0$

$\Rightarrow k(k - 8) - 4(k - 8) = 0$

$\Rightarrow (k - 8)(k - 4) = 0$

Either $k - 8 = 0$, then $k = 8$

or $k - 4 = 0$, then $k = 4$

But $k - 4 \neq 0$

$k \neq 4$

$k = 8$

Question 6.

Find the value(s) of m for which each of the following quadratic equation has real and equal roots:

(i) $(3m + 1)x^2 + 2(m + 1)x + m = 0$

(ii) $x^2 + 2(m - 1)x + (m + 5) = 0$

Solution:

(i) $(3m + 1)x^2 + 2(m + 1)x + m = 0$

Here $a = 3m + 1$, $b = 2(m + 1)$, $c = m$

$\therefore$ $D = b^2 - 4ac$

$= [2(m + 1)]^2 - 4 \times (3m + 1)(m)$

$= 4(m^2 + 2m + 1) - 12m^2 - 4m$

$= 4m^2 + 8m + 4 - 12m^2 - 4m$

$= -8m^2 + 4m + 4$

$\therefore$ Roots are equal.

$\therefore$ $D = 0$

$\Rightarrow -8m^2 + 4m + 4 = 0$

$\Rightarrow 2m^2 - m - 1 = 0$ (Dividing by 4)

$\Rightarrow 2m^2 - 2m + m - 1 = 0$

$\Rightarrow 2m(m - 1) + 1(m - 1) = 0$

$\Rightarrow (m - 1)(2m + 1) = 0$

Either $m - 1 = 0$, then $m = 1$

or $2m + 1 = 0$, then $2m = -1$

$\Rightarrow m = -\frac{1}{2}$

Question 7.

Find the values of k for which each of the following quadratic equation has equal roots:

(i) $9x^2 + kx + 1 = 0$

(ii) $x^2 - 2kx + 7k - 12 = 0$

Also, find the roots for those values of k in each case.

Solution:

(i) $9x^2 + kx + 1 = 0$

Here a = 9, b = k, c = 1

$\therefore$ $D = b^2 - 4ac$

$= k^2 - 4 \times 9 \times 1 = k^2 - 36$

$\because$ Roots are equal.

$\therefore$ $D = 0$

$\Rightarrow k^2 - 36 = 0 \Rightarrow (k + 6)(k - 6) = 0$

Either $k + 6 = 0$, then $k = -6$

$k - 6 = 0$, then $k = 6$

$\therefore$ $k = 6, -6$

(a) If $k = 6$, then

$9x^2 + 6x + 1 = 0$

$\Rightarrow (3x)^2 + 2 \times 3x \times 1 + (1)^2 = 0$

$\Rightarrow (3x + 1)^2 = 0$

$\therefore$ $3x + 1 = 0 \Rightarrow 3x = -1$

$$x = -\frac{1}{3}, -\frac{1}{3}.$$

(b) If $k = -6$, then

$9x^2 - 6x + 1 = 0$

$\Rightarrow (3x)^2 - 2 \times 3x \times 1 + (1)^2 = 0$

$\Rightarrow (3x - 1)^2 = 0 \Rightarrow 3x - 1 = 0$

$\Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$

$$x = \frac{1}{3}, \frac{1}{3}$$

(ii) $x^2 - 2kx + 7k - 12 = 0$

Here $a = 1$, $b = -2k$, $c = 7k - 12$

$\therefore$ $D = b^2 - 4ac$

$= (-2k)^2 - 4 \times 1 \times (7k - 12)$

$= 4k^2 - 4(7k - 12)$

$= 4k^2 - 28k + 48$

$\because$ Roots are equal

$\therefore$ $D = 0$

$\Rightarrow 4k^2 - 28k + 48 = 0$

$\Rightarrow k^2 - 7k + 12 = 0$

$\Rightarrow k^2 - 3k - 4k + 12 = 0$

$\Rightarrow k(k - 3) - 4(k - 3) = 0$

$\Rightarrow (k - 3)(k - 4) = 0$

Either $k - 3 = 0$, then $k = 3$

or $k - 4 = 0$, then $k = 4$

(a) If $k = 3$, then

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{4k \pm \sqrt{0}}{2 \times 1} = \frac{4 \times 3}{2} = \frac{12}{2} = 6$$

$x = 6, 6$

(b) If $k = 4$, then

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-2 \times 4) \pm \sqrt{0}}{2 \times 1} = \frac{+8}{2} = 4$$

$\therefore$ $x = 4, 4$

Question 8.
Find the value(s) of p for which the quadratic equation $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots.

Solution:

The quadratic equation given is $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$

Comparing with $ax^2 + bx + c = 0$, we have

$a = 2p + 1,\ b = -(7p + 2),\ c = (7p - 3)$

$D = b^2 - 4ac \Rightarrow 0 = [-(7p + 2)]^2 - 4(2p + 1)$
$(7p - 3)$

$0 = 49p^2 + 4 + 28p - 4(14p^2 - 6p + 7p - 3)$

$0 = 49p^2 + 4 + 28p - 56p^2 - 4p + 12$

$0 = -7p^2 + 24p + 16$

$0 = -7p^2 + 28p - 4p + 16$

$0 = -7p(p - 4) - 4(p - 4)$

$0 = (-7p - 4)(p - 4)$

$\Rightarrow -7p - 4 = 0$ or $p - 4 = 0$

Hence, the vaue of $p = \frac{-4}{7}$ or $p = 4$

Question 9.

If – 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution:

-5 is a root of the quadratic equation

$2x^2 + px - 15 = 0$, then

$\Rightarrow 2(5)^2 - p(-5) - 15 = 0$

$\Rightarrow 50 - 5p - 15 = 0$

$\Rightarrow 35 - 5p = 0$

$$\Rightarrow 5p = 35 \Rightarrow p = \frac{35}{5} = 7$$

$p(x^2 + x) + k = 0$ has equal roots

$\Rightarrow px^2 + px + k = 0$

$\Rightarrow 7x^2 + 7x + k = 0$

Here, $a = 7, b = 7, c = k$

$b^2 - 4ac = (7)^2 - 4 \times 7 \times k$

$= 49 - 28k$

$\because$ Roots are equal

$\therefore b^2 - 4ac = 0$

$\Rightarrow 49 - 28k = 0$

$$\Rightarrow 28k = 49 \Rightarrow k = \frac{49}{28} = \frac{7}{4}$$

$$\therefore k = \frac{7}{4}$$

Question 10.

Find the value(s) of p for which the equation $2x^2 + 3x + p = 0$ has real roots.

Solution:

$2x^2 + 3x + p = 0$

Here, a = 2, b = 3, c = p

$b^2 - 4ac = (3)^2 - 4 \times 2 \times p = 9 - 8p$

$\because$ Roots are real

$\therefore b^2 - 4ac \geq 0 \Rightarrow 9 - 8p \geq 0$

$$9 \geq 8p \Rightarrow 8p \leq 9 \Rightarrow p \leq \frac{9}{8}$$

Question 11.

Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.

Solution:

$x^2 + kx + 4 = 0$

Here, a = 1, b = k, c = 4

$b^2 - 4ac = k^2 - 4 \times 1 \times 4$

$= k^2 - 16$

$\because$ Roots are real and positive.

$\therefore k^2 - 16 \geq 0 \Rightarrow k^2 \geq 16$

$\Rightarrow k \geq 4 \Rightarrow k = 4$

Question 12.

Find the values of p for which the equation $3x^2 - px + 5 = 0$ has real roots.

Solution:

$3x^2 - px + 5 = 0$

Here, a = 3, b = -p, c = 5

$\therefore b^2 - 4ac = (-p)^2 - 4 \times 3 \times 5$

$= p^2 - 60$

$\therefore$ Roots are real

$\therefore b^2 - 4ac \geq 0$

$\therefore p^2 - 60 \geq 0 \Rightarrow p^2 \geq 60$

$\Rightarrow p \geq \pm\sqrt{60} = \pm 2\sqrt{15}$

$\therefore p \leq -2\sqrt{15}$ or $p \geq 2\sqrt{15}$