Chapter 8 Matrices Ex 8.2

Question 1.

Given that M = $[2102]$ and N = $[2{-}102]$,find M + 2N

Solution:

$$M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\therefore M + 2N = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 0+0 \\ 1-2 & 2+4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}$$

Question 2.

If A = $[2{-}301]$ and B = $[0{-}213]$

find 2A – 3B

Solution:

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

$$\therefore 2A - 3B = 2\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix} = \begin{bmatrix} 4-0 & 0-3 \\ -6+6 & 2-9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$$

Question 3.

If A = $[1243]$ and B = $[1321]$

Compute 3A + 4B

Solution:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$3A + 4B = 3\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 12 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 & 12+8 \\ 6+12 & 9+4 \end{bmatrix} = \begin{bmatrix} 7 & 20 \\ 18 & 13 \end{bmatrix}$$

Question 4.

Given A = [1243] and B = [−4−3−1−2]

(i) find the matrix 2A + B

(ii) find a matrix C such that C + B = [0000]

Solution:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

$$(i) \quad 2A + B = 2\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4 & 8-1 \\ 4-3 & 6-2 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & 4 \end{bmatrix}$$

$$(ii) \quad C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-(-4) & 0-(-1) \\ 0-(-3) & 0-(-2) \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

Question 5.

A = [1−223] and B = [−21−12] , C = [023−1]

Find A + 2B – 3C

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$\therefore A + 2B - 3C$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4-0 & 2-2-9 \\ -2+2-6 & 3+4+3 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$

Question 6.

If A = [01−12] and B = [1−121]

Find the matrix X if :

(i) 3A + X = B

(ii) X – 3B = 2A

Solution:

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

(i) 3A + X = B

⇒ X = B − 3A

$$X = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1-0 & 2+3 \\ -1-3 & 1-6 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -4 & -5 \end{bmatrix}$$

(*ii*) $X - 3B = 2A \Rightarrow x = 2A + 3B$

$$X = 2\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3 & -2+6 \\ 2-3 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$$

Question 7.

Solve the matrix equation

$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 2 \\ 4 & 6 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix} = 3X$$

$$\therefore X = \frac{1}{3}\begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Question 8.

If $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 0 \\ 2 & -3 \end{bmatrix}$, find the matrix M

Solution:

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$$

2M =

$$3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 6-4 \\ 0-(-2) & -9-3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$$

$$\therefore \mathbf{M} = \frac{1}{2}\begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$$

(Dividing by 2)

Question 9.

A = $\begin{bmatrix} 2 & 2 \\ -6 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Find the matrix X such that A + 2X = 2B + C

Solution:

$A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

let $X = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$

$$A + 2X = 2B + C$$
$$2X = 2B + C - A$$

$$2\begin{bmatrix} x & y \\ z & t \end{bmatrix} = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6+4-2 & 4+0+6 \\ 8+0-2 & 0+2-0 \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$\therefore 2\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$

Question 10.

Find X and Y if X + Y = [7205] and X – Y = [3003]

Solution:

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \text{.....(i)}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{.....(ii)}$$

Adding (*i*) and (*ii*) we get, $2x = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore \quad x = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Subtracting (*ii*) from (*i*), $2y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow 2y = \begin{bmatrix} 7-3 & 0-0 \\ 2-0 & 5-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\therefore \quad y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Hence $x = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}, y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Question 11.

If 2[354x]+[10y1]=[71005] Find the values of x and y

Solution:

$$2\begin{bmatrix}3 & 4\\ 5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$$

$$\begin{bmatrix}6 & 8\\ 10 & 2x\end{bmatrix}+\begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}6+1 & 8+y\\ 10+0 & 2x+1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$$

Comparing the corresponding elements,

$8 + y = 0$ then $y = -8$

$2x + 1 = 5$ then $2x = 5 - 1 = 4 \Rightarrow x = 2$

Hence $x = 2, y = -8$

Question 12.

If 2[354x]+[10y1]=[z1005] Find the values of x and y

Solution:

$$2\begin{bmatrix}3 & 4\\ 5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix}=\begin{bmatrix}z & 0\\ 10 & 5\end{bmatrix}$$

$$\begin{bmatrix}6 & 8\\ 10 & 2x\end{bmatrix}+\begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix}=\begin{bmatrix}z & 0\\ 10 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}6+1 & 8+y\\ 10+0 & 2x+1\end{bmatrix}=\begin{bmatrix}z & 0\\ 10 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}7 & 8+y\\ 10 & 2x+1\end{bmatrix}=\begin{bmatrix}z & 0\\ 10 & 5\end{bmatrix}$$

Comparing,

$2x + 1 = 5 \Rightarrow 2x = 5 - 1 = 4$

$\therefore x = \frac{4}{2} = 2$

$8 + y = 0, \Rightarrow y = -8$

$z = 7$

Hence $x = 2, y = -8, z = 7$

Question 13.

If [5−12y+1]−2[132x−1−2]=[3−7−82] Find the values of x and y

Solution:

$$\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2x-1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - \begin{bmatrix} 2 & 4x-2 \\ 6 & -4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5-2 & 2-4x+2 \\ -1-6 & y+1+4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 4-4x \\ -7 & y+5 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

Comparing the corresponding terms, we get

$4 - 4x = -8 \Rightarrow -4x = -8 - 4$

$\Rightarrow -4x = -12 \Rightarrow x = \dfrac{-12}{-4} = 3$

and $y + 5 = 2 \Rightarrow y = 2 - 5 = -3$

$\therefore\ x = 3, y = -3$

Question 14.

If $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

Find the value of a,b and c

Solution:

$$\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 2-2-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+1 & b+2 \\ 7 & -c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

Comparing the corresponding elements :

$a + 1 = 5 \Rightarrow a = 4$

$b + 2 = 0 \Rightarrow b = -2$

$-c = 3 \Rightarrow c = -3$

Question 15.

If A = $\begin{bmatrix} 2 & -3 \\ a & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 7 \\ 3 & b \end{bmatrix}$, C = $\begin{bmatrix} c & -1 \\ 9 & -11 \end{bmatrix}$ and 5A + 2B = C, find the values of a, b, c

Solution:

$A = \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, $C = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

and $5A + 2B = C$

$$\Rightarrow 5\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix} + 2\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 5a \\ -15 & 25 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 14 & 2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10-4 & 5a+6 \\ -15+14 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 5a+6 \\ -1 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

Comparing each term

$5a + 6 = 9 \Rightarrow 5a = 9 - 6 = 3$

$\Rightarrow a = \frac{3}{5} \quad \Rightarrow 25 + 2b = -11$

$\Rightarrow 2b = -11 - 25 = -36 \Rightarrow b = -\frac{36}{2} = -18$

$c = 6$

Hence $a = \frac{3}{5}$, $b = -18$ and $c = 6$